Analysis of the Perfect Bayesian Equilibrium in Hearing Game

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Abstract: The main purpose of this paper is to analyze the perfect Bayesian equilibrium in hearing game by methods of signal transmission game. Firstly, it is shown that there is an equilibrium in hearing game, where the forerunner’s action has no effect on the outcome. Then it is analyzed that there is a perfect Bayesian equilibrium in the game, which can provide signal. Finally, a closed rule of hearing game is discussed.

1. Introduction

The hearing system is a new system that introduces public participation in social decision-making. In recent years, the hearing system has become a hot topic. It has been proved that the extensive holding of hearings reflects the progress of social democracy construction. It is becoming an important part of administrative decision-making and system construction procedure.

Hearings are the main way for citizens to participate in politics. This needs to ensure that citizens can participate in the hearing effectively, that is to create conditions for citizens to supervise the government and fully ensure the realization of civil rights and obligations. The citizen participation in the hearing can protect the status of citizens, which has practical significance for scientific.

When citizens participate in the hearing, the government can learn relevant information from the citizens. At the same time, citizens can also help the government to make decisions and supervise the policy-making, so that the government can form a scientific point of view, make decisions in line with the public interest, and promote the scientific rationality of the hearing.

Game theory[1-3] is a mathematical theory and method to study the phenomenon of struggle or competition. Signaling game[4-6] is a game in which there are two participants, the sender sends private information and the receiver makes decisions based on the information of the sender. Therefore, it is very effective to use the method of signaling game to study the hearing game.

In this paper, by methods of signal transmission game, the perfect Bayesian equilibrium in hearing game is analyzed. Firstly, it is shown that there is an equilibrium in hearing game, where the forerunner’s action has no effect on the outcome. Then it is analyzed that there is a perfect Bayesian equilibrium in the game, which can provide signal. Finally, a closed rule of hearing game is discussed.

2. Hearing Game

For convenience, we simply describe the hearing game as a signaling game in the following forms.

There are two players in hearing game. One is player government, and the other is player citizen. Player government acts first. He suggests a policy a_1 to player citizen. After observing a_1, player citizen plays action. The decision-making content is a policy a_2. The outcome of a_2 is $x = a_2 + \omega$, where ω is a random variable that is uniform distribution on the interval $[0,1]$, that is $\omega \sim U(0,1)$. Player government knows ω, but player citizen doesn't. The preferences of both players are quadratic with respect to citizen's happiness point $x = 0$ and player government's happiness point $x = x_G$, $x_G \in (0,1)$:

$$u_1(x) = -(x - x_G)^2 \text{ and } u_2(x) = -x^2.$$

Firstly, we will show that there is an equilibrium in hearing game, where player government's action has no effect on the outcome.

Consider the following strategy profile and beliefs:

Player government chooses $a_1 = 0$ regardless of ω.

Player citizen believes ω is uniform on [0,1] after any announcement a_1. He chooses $a_2 = -\frac{1}{2}$ after all a_1.

Thus, the beliefs are clearly compatible with Bayesian updating after all a_1. Given his beliefs, player citizen chooses $a_2(a_1)$ to maximize $E_{\omega|a_1}[-(a_2+\omega)^2]$. This expression is maximized for $a_2 = -E(\omega)$ by standard statistics. Hence $a_2 = -\frac{1}{2}$ is a best response.

As player government's action has no effect on the outcome, his strategy is also a best response.

Then we will prove that there is a perfect Bayesian equilibrium in hearing game, which can provide signal.

Consider the following strategies:

Player government announces $a_1 < 0$, i.e. he chooses a_1 at random from a distribution with full support on $(-\infty, 0)$, if $\omega \in [0, 2x_G + \frac{1}{2}]$;

He announces $a_1 \geq 0$ if $\omega \in (2x_G + \frac{1}{2}, 1]$.

Player citizen chooses $a_2 = \begin{cases} -\frac{1}{2}(2x_G + \frac{1}{2}) & a_1 < 0 \\ -\frac{1}{2}(2x_G + \frac{3}{2}) & a_1 \geq 0 \end{cases}$.

Let $\omega^* = 2x_G + \frac{1}{2}$. Suppose government plays the strategy above. Then player citizen's best response is to choose $a_2(a_1)$ to maximize $-E[(a_2+\omega)^2 \mid a_1]$, Thus $a_2(a_1) = -E(\omega \mid a_1)$.

We know that $E(\omega \mid a_1 < 0) = \frac{\omega^*}{2}$, since beliefs are $\omega \sim U[0, \omega^*]$; $E(\omega \mid a_1 \geq 0) = \frac{1+\omega^*}{2}$. So the strategy for player citizen is a best response.

If player government observes $\omega \in [0, \omega^*]$, his payoffs to announcing

$$\begin{cases} -\{x_G - [-\frac{1}{2}(2x_G + \frac{1}{2}) + \omega]\}^2 & a_1 \leq 0 \\ -\{x_G - [-\frac{1}{2}(2x_G + \frac{3}{2}) + \omega]\}^2 & a_1 > 0 \end{cases}.$$

The first expression is larger and player government is playing a best response if $(2x_G + \frac{1}{4} - \omega)^2 \leq (2x_G + \frac{3}{4} - \omega)^2$ if and only if $\omega \leq \frac{1}{2}[(2x_G + \frac{1}{4}) + (2x_G + \frac{3}{4})] = 2x_G + \frac{1}{2}$. The same argument shows $a_1 \geq 0$ is a best response for player government if $\omega \in (\omega^*, 1]$. Hence the strategies is a perfect Bayesian equilibrium in the game.

Finally, we will discuss a closed rule of hearing game.

Depending on the values of the parameters, the closed rule game may have an equilibrium, a perfectly revealing equilibrium and other partially revealing equilibria.

For any $a_0 \in (-1,0)$, we can construct an equilibrium where status quo a_0 is always chosen regardless of the proposal a_1. Choose a_1 with $a_0^2 + a_0 \le a_1^2 + a_1$ and let the strategies and beliefs be:

Player government always announces a_1.

Player citizen chooses the status quo a_0 over a_1 or any $a_1' \ne a_1$.

If a_1 was announced believe $\omega \sim U[0,1]$.

If $a_1' \le a_0$, $a_1' \ne a_1$ was announced believe $\omega = 0$.

If $a_1' > a_0$, $a_1' \ne a_1$ was announced believe $\omega = 1$.

Player government can not affect the outcome, so he is playing a best response. After a_1 is announced, player citizen's utilities from his two choices are

$$Eu_2(a_0) = -E(a_0+\omega)^2 = -[a_0^2 + 2a_0E(\omega) + E(\omega^2)] = -(a_0^2 + a_0 + \frac{1}{3})$$

$$Eu_2(a_1) = -E(a_1+\omega)^2 = -[a_1^2 + 2a_1E(\omega) + E(\omega^2)] = -(a_1^2 + a_1 + \frac{1}{3}).$$

So given the a_1 above, choosing a_0 is optimal.

When $a_1' \le a_0$, the somewhat unreasonable sounding, but allowable, beliefs are that $a_1' + \omega = a_1' \le a_0 < 0$, so a_0 is preferred. The belief $\omega = 1$ when $a_1' > a_0$ is announced gives us $0 < a_0 + \omega < a_1 + \omega$, so a_0 is preferred in that case as well.

When $a_0 \le -x_G - 1$ or $a_0 > x_G$ we can construct a fully revealing equilibrium where the announcement $a_1(\omega)$ reveals the value of ω. Let the strategies be:

Player government announces $a_1(\omega) = x_G - \omega$.

Player citizen chooses a_1 over a_0.

Here the government gets its most preferred outcome, so is satisfied with the strategy. The citizen is happy to choose a_1 as the utility from doing, so $-x_G^2$ is greater than $-(a_0+\omega)^2$ for any ω.

There may also be semi-separating equilibria. For example, if $a_0 \in (x_G - 1, x_G)$, we have the equilibrium:

Player government proposes a_0 for $\omega \in [0, x_G - a_0]$, $x_G - \omega$ for $\omega \in (x_G - a_0, 1]$.

$$\text{i.e. } \begin{cases} a_0 & \omega \in [0, x_G - a_0] \\ x_G - \omega & \omega \in (x_G - a_0, 1] \end{cases}.$$

Player citizen choose a_1 whenever $a_1 \in (x_G - 1, a_0]$, and chooses a_0 when $a_1 > a_0$, and chooses something when $a_1 < x_G - 1$.

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